2/ $e^{2u_p-u_{2p}} = \dfrac{(e^{u_p})^2}{e^{u_{2p}}} = \dfrac{\left(\dfrac{p!}{p^{p+\frac{1}{2}}}.e^p\right)^2}{\dfrac{(2p)!}{(2p)^{(2p)+\frac{1}{2}}}.e^{(2p)}} = \dfrac{\dfrac{(p!)^2}{p^{2p+1}}e^{2p}}{\dfrac{(2p)!}{(2p)^{(2p)+\frac{1}{2}}}.e^{(2p)}}$

$= \dfrac{(p!)^2(2p)^{(2p)}\sqrt{2p}}{p^{2p+1}(2p)!} = \dfrac{(p!)^2(2^p)^2p^{2p}\sqrt{2p}}{p^{2p+1}(2p)!}$

$= \dfrac{\sqrt{2}}{\sqrt{p}}\dfrac{(2^p.p!)^2}{(2p)!} = \dfrac{(2p+1)(2p+3)}{\sqrt{2}\sqrt{p}} \times \dfrac{2}{(2p+1)(2p+3)}.\dfrac{(2^p.p!)^2}{(2p)!}$

$= \dfrac{(2p+1)(2p+3)}{\sqrt{2}\sqrt{p}} \times v_{2p}.$

3/ • $\lim\limits_{p\to+\infty} u_{2p} = \lim\limits_{p\to+\infty} u_p = L \Rightarrow \lim\limits_{p\to+\infty} e^{2u_p-u_{2p}} = e^{2L-L} = e^L$ car la fonction exponentielle est continue sur IR en particulier en L.

• $\lim\limits_{p\to+\infty} \dfrac{(2p+1)(2p+3)}{\sqrt{2}\sqrt{p}} \times v_{2p} = \lim\limits_{p\to+\infty} \dfrac{(2p+1)(2p+3)}{\sqrt{2}\sqrt{p}\,(2p)^{\frac{3}{2}}} \times (2p)^{\frac{3}{2}}v_{2p}$

$= \lim\limits_{p\to+\infty} \dfrac{(2p+1)(2p+3)}{4p^2} \times \lim\limits_{p\to+\infty} \left[(2p)^{\frac{3}{2}}v_{2p}\right]$

$= \lim\limits_{p\to+\infty} \dfrac{4p^2}{4p^2} \times \sqrt{2\pi} = \sqrt{2\pi}$

Ainsi $e^L = \sqrt{2\pi} \Leftrightarrow L = \text{Log}\sqrt{2\pi}$.